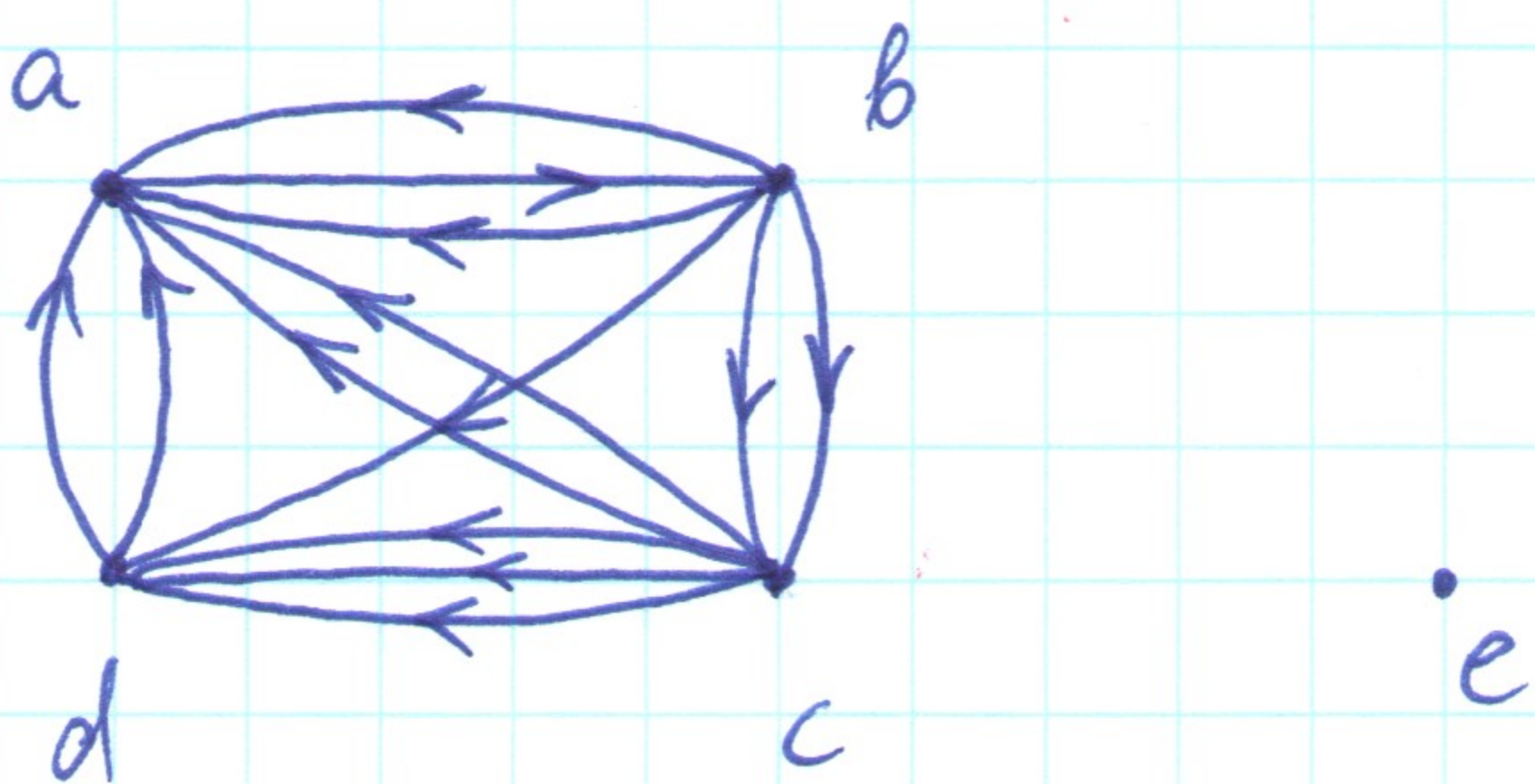


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$$|V| = 5$$

$$|E| = 13$$

in-degree

$$\text{deg}^-(a) = 6$$

$$\text{deg}^-(b) = 1$$

$$\text{deg}^-(c) = 2$$

$$\text{deg}^-(e) = 0$$

$$\text{deg}^-(d) = 4$$

out-degree

$$\text{deg}^+(a) = 1$$

$$\text{deg}^+(b) = 5$$

$$\text{deg}^+(c) = 5$$

$$\text{deg}^+(e) = 0$$

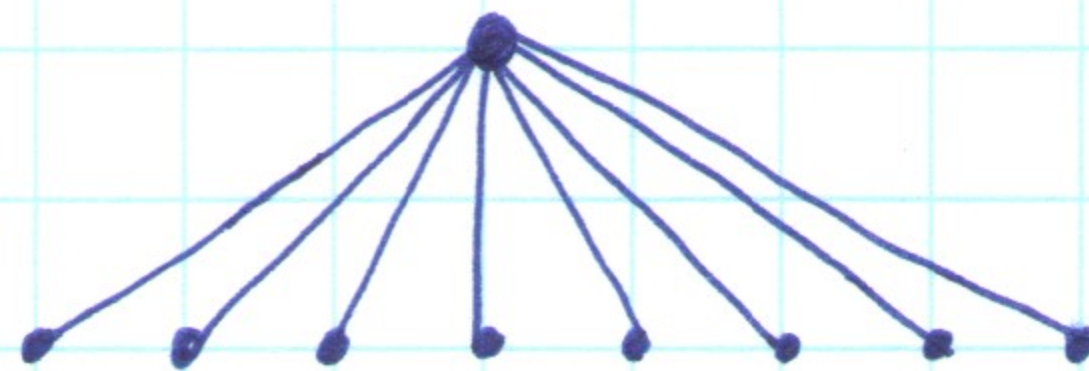
$$\text{deg}^+(d) = 2$$

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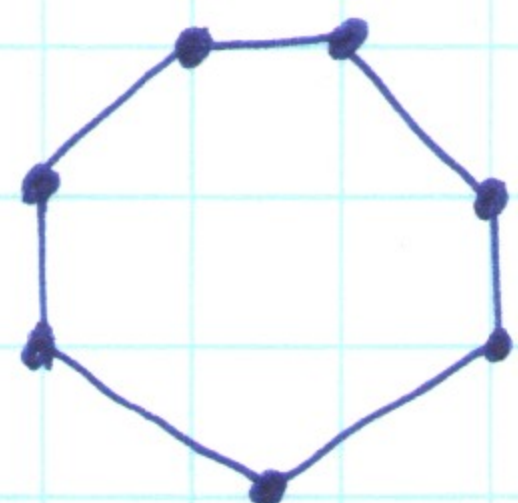
a) K_7



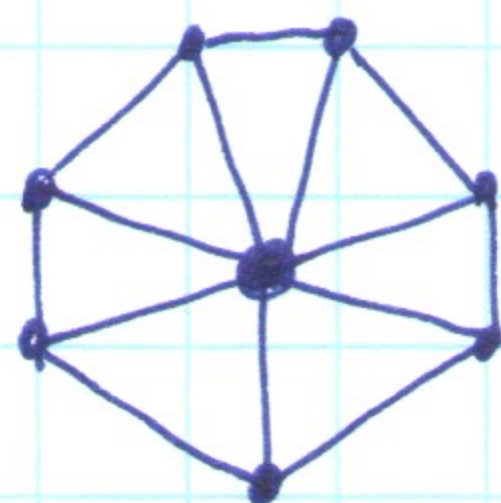
b) $K_{1,8}$



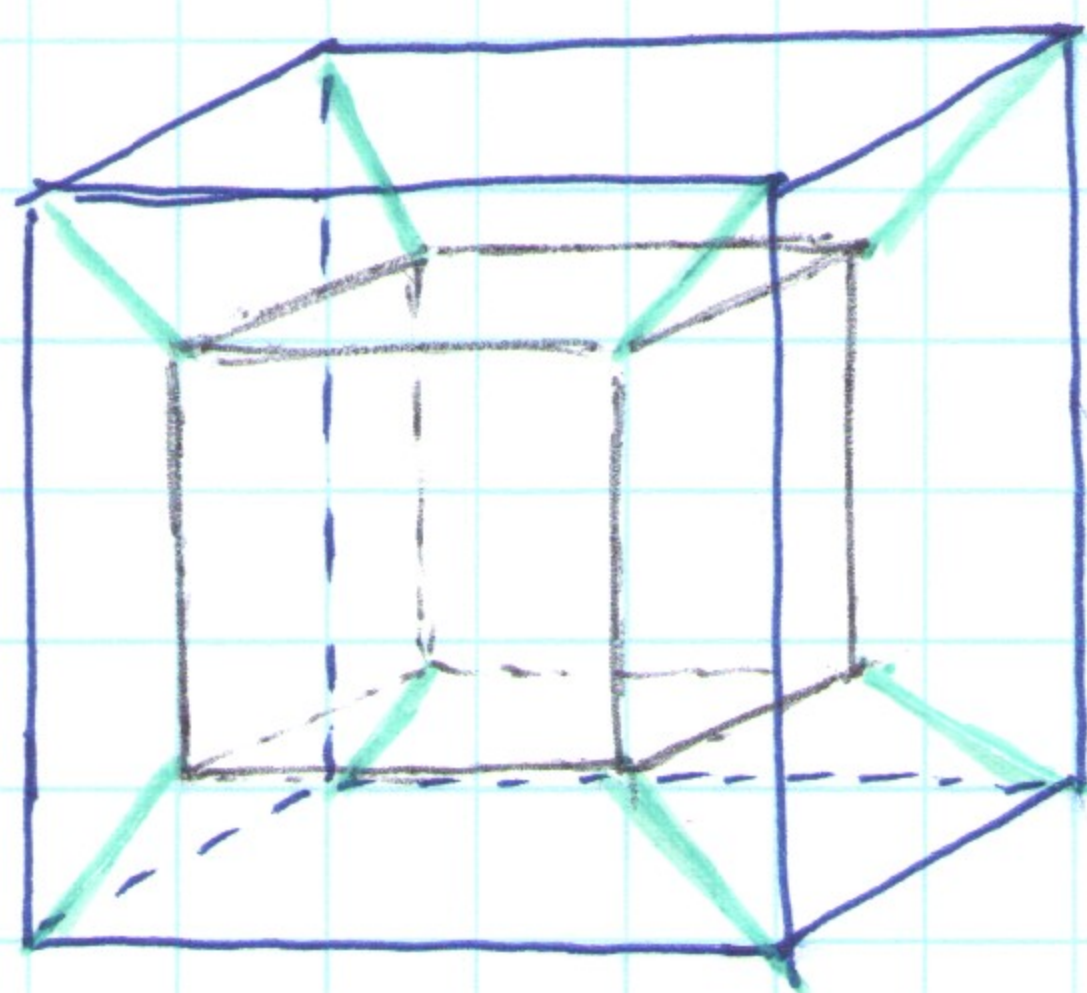
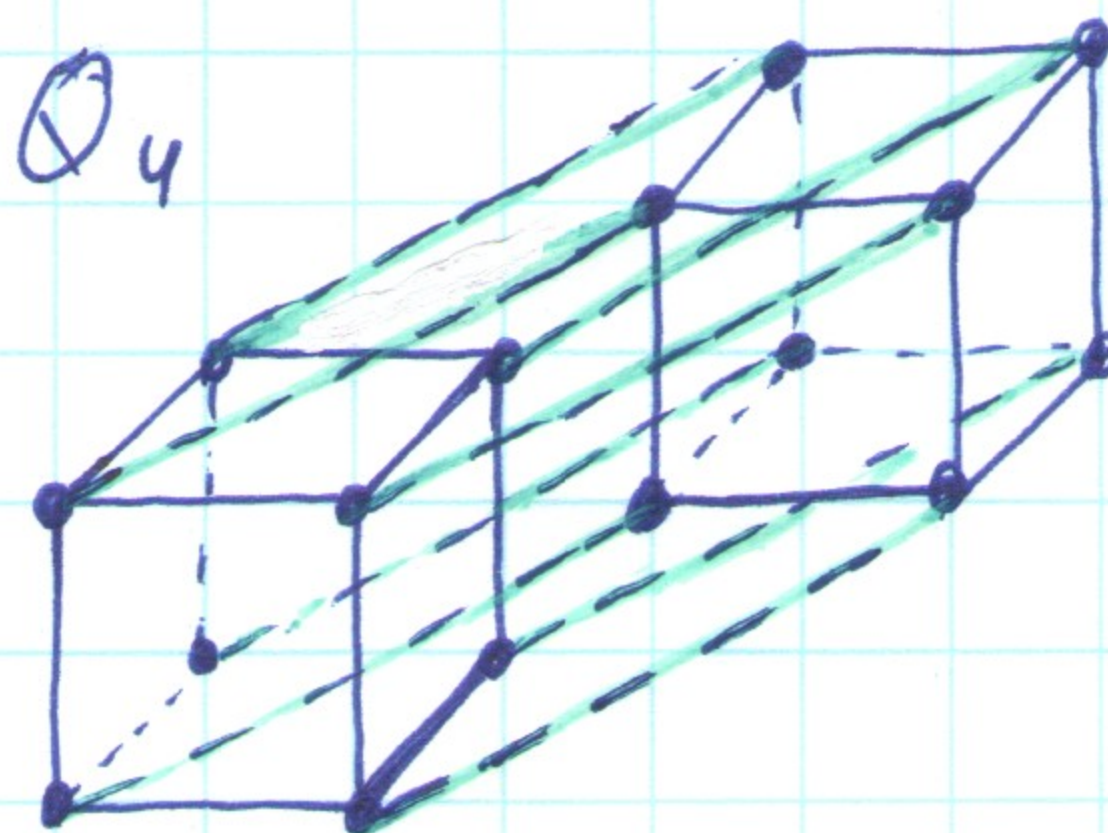
d) C_7



e) W_7

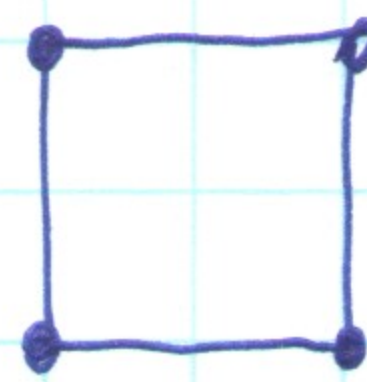


f) Q_4

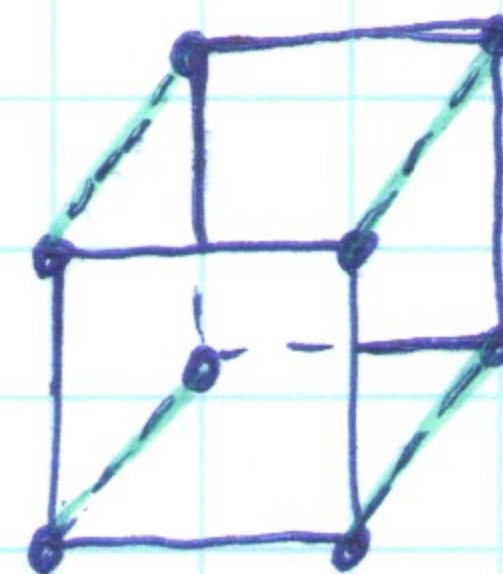


or

this is similar to transition from Q_2 to Q_3 :



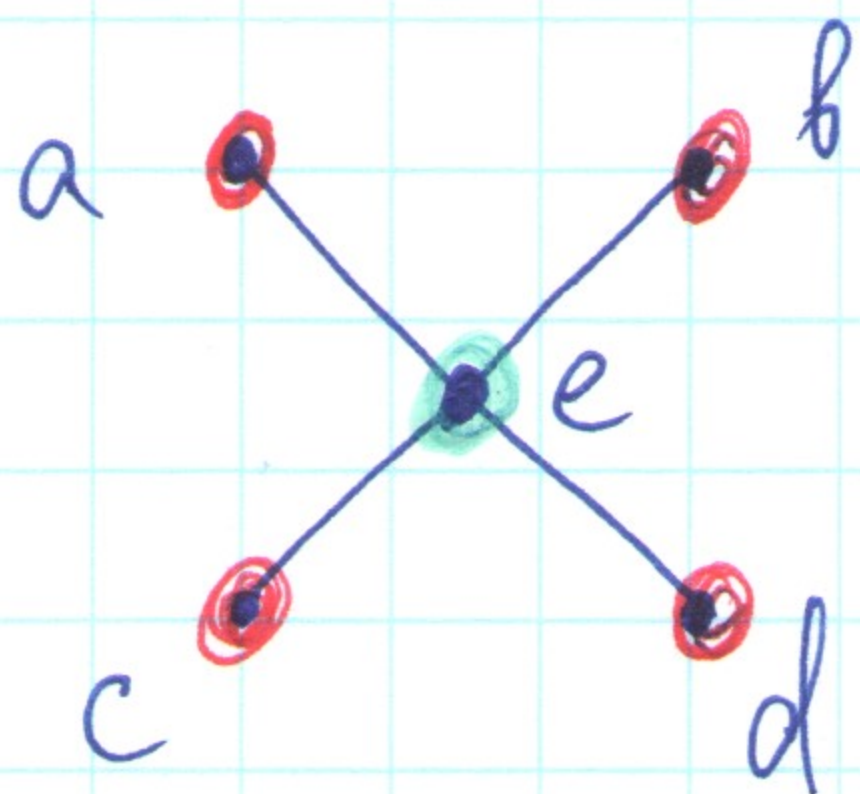
Q_2



Q_3

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determine whether the graph is bipartite

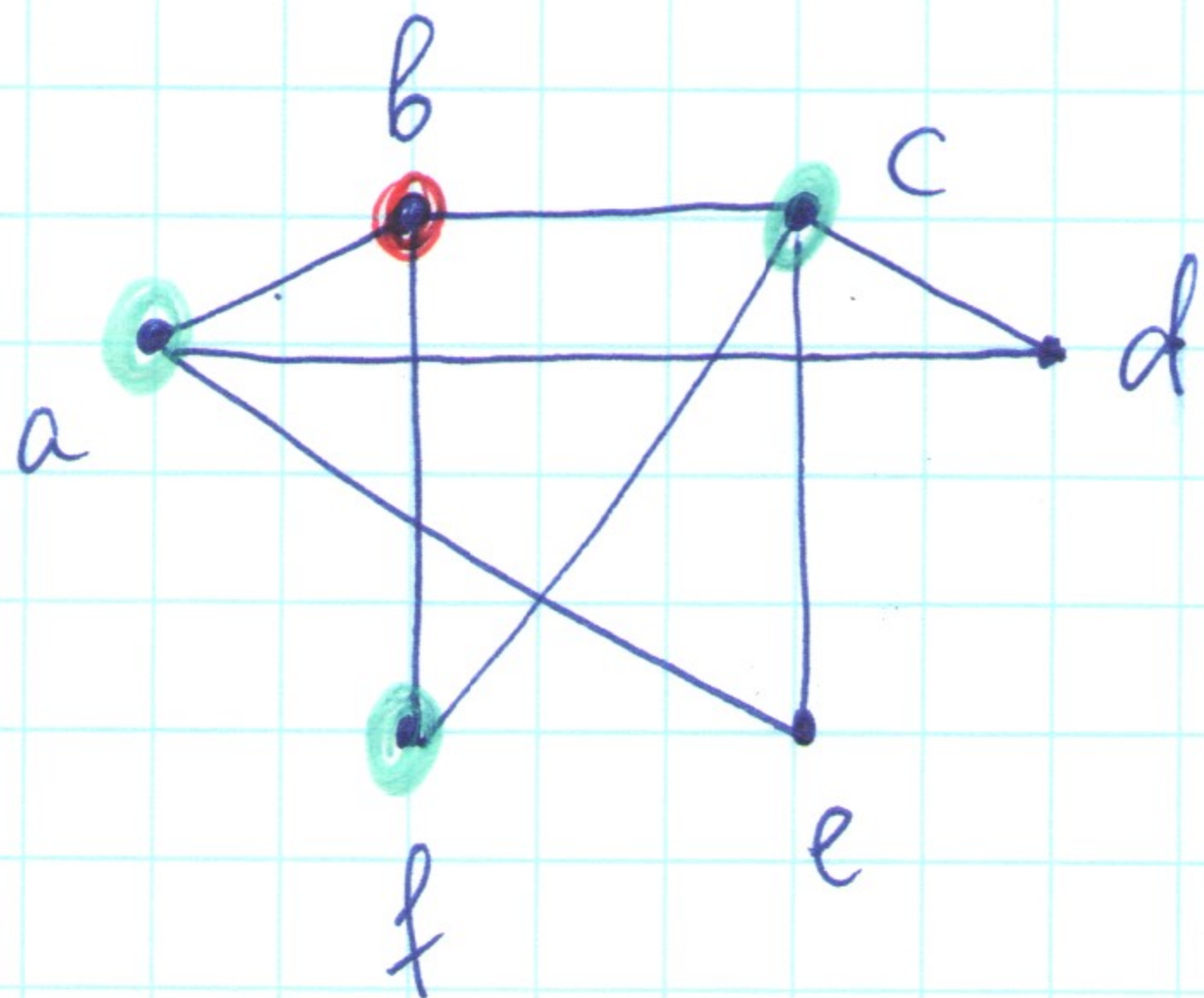


- start by marking a with red
- then mark e green
- then mark c, b, and d red

The graph is bipartite

$V_1 = \{a, b, c, d\}$, $V_2 = \{e\}$

determine whether the graph is bipartite.



- start with b marked red

- then mark a, c, f green

- then mark e, d, STOP!

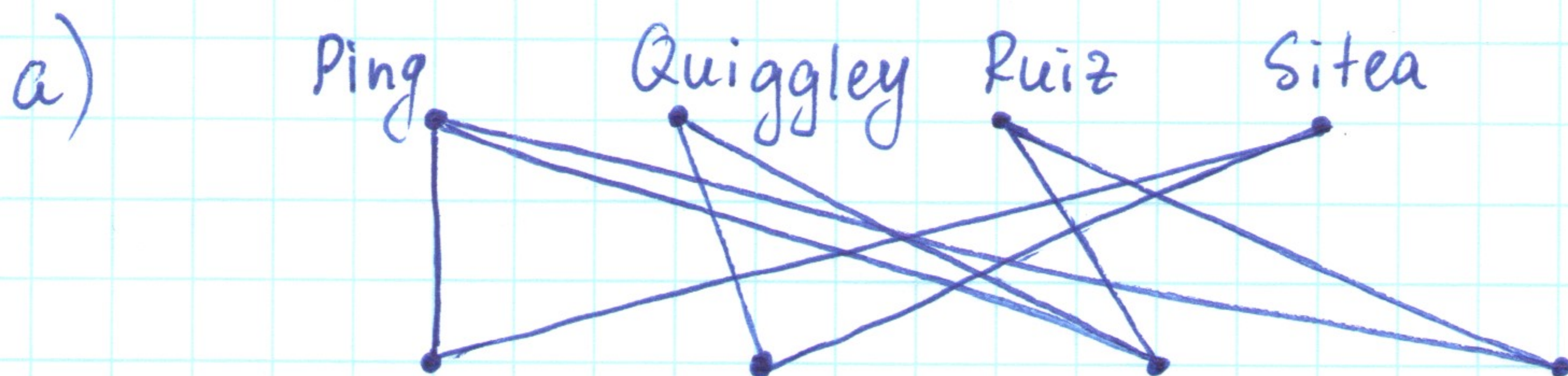
c and f are adjacent, and have the same color!, hence

The graph is not bipartite

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support group: 4 people

areas: hardware, software, networking, wireless.



Hardware Software Networking Wireless

b) according to Hall's theorem a bipartition exists iff

$$|N(A)| \geq |A| \text{ for } \forall A \subseteq V_1$$

$V_1 = \{ \text{Ping, Quiggley, Ruiz, Sitea} \}$

we will have to check this inequality for all possible subsets of V_1

- 1) let $A = \{ \text{Ping} \}$, hence $|A| = 1$ and $|N(A)| = 3 > 1$ ✓ 371
- 2) let $A = \{ \text{Quiggley} \}$, hence $|A| = 1$ and $|N(A)| = 2 > 1$ ✓ 271
- 3) let $A = \{ \text{Ruiz} \}$, hence $|A| = 1$ and $|N(A)| = 2 > 1$ ✓ 271
- 4) let $A = \{ \text{Sitea} \}$, hence $|A| = 1$ and $|N(A)| = 2 > 1$ ✓ 271
- 5) let $A = \{ \text{Ping, Quiggley} \}$, hence $|A| = 2$ and $|N(A)| = 3 + 2 = 5 > 2$ ✓ 572

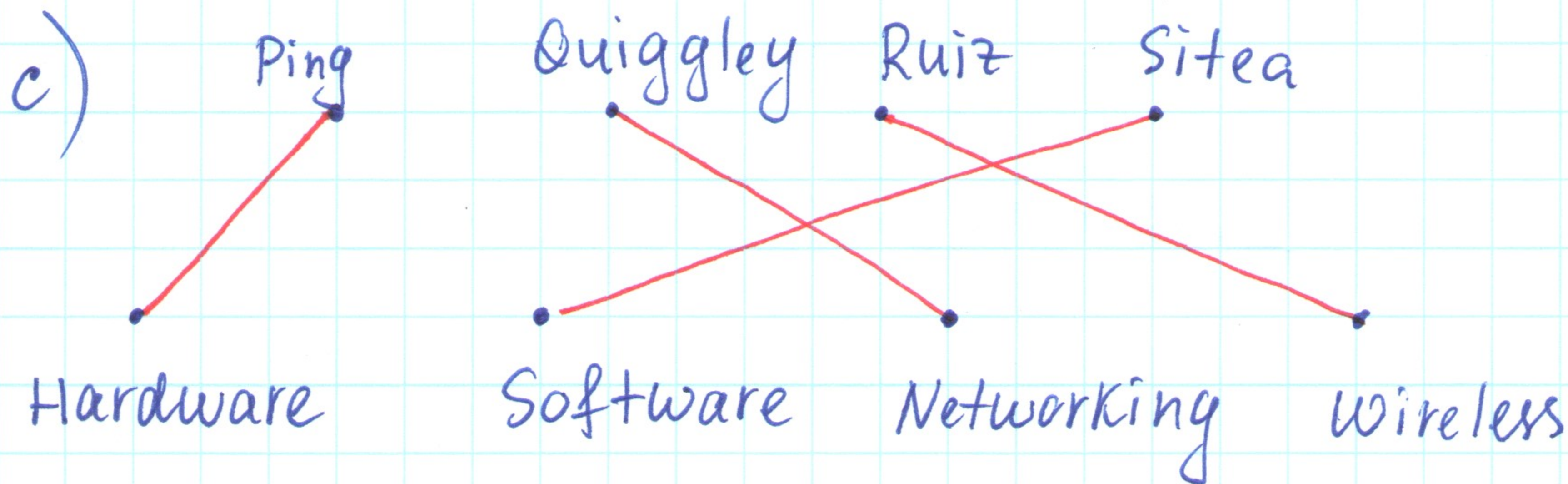
6) let $A = \{ \text{Ping, Ruiz} \}$ hence $|A| = 2$ and
 $|N(A)| = 2 + 2 = 4 > 2$ ✓ $4 > 2$

7) let $A = \{ \text{Ping, Sitea} \}$ hence $|A| = 2$ and
 $|N(A)| = 3 + 2 = 5 > 2$ ✓ $5 > 2$

8) let $A = \{ \text{Quiggley, Ruiz} \}$, hence $|A| = 2$ and
 $|N(A)| = 2 + 2 = 4 > 2$ ✓ $4 > 2$

and so forth there will be 7 more checks

and all of them will yield true inequalities
 (because each person in a group has 2 or more qualifications,
 so with addition of a person, the neighborhood increases
 by 2 or 3)



Note that this is one of a number of possible assignments.